Emergence of Modern Ballistics:
The Problem of Exchange of Knowledge in 18th Century Europe

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Abstract: the development of the Ballistics can play an important role in the history of science and technology when we consider the relation between the empirical study and theoretical one. In this study, we chiefly compare two mathematicians' studies and know the difference between these ways of analyzing real and highly political objects. There are several possibilities of exchange of Scientific Knowledge although usually the historian of mathematics has a tendency to underline the discontinuation between British mathematics and Continental one in the 18th Century.

1. Purpose and Range of the Study

Here I indicate purpose and range of my study. It is a kind of complement to the traditional view and way of historiography in 17th-18th Century. We have some typical cases of conflicts: 1) Newton and Leibniz's priority race on the discovery of calculus and 2) George Barkley and Continental Mathematicians discussion over the existence and utilization of infinitesimal small quantity.

And I treat my purpose and range more profoundly. There is one question: is there any other way in order to represent a new historiography of 18th Century? Main appropriate material is an example that can express the reception and conflict and exchange of Knowledge in this era, c'est-a-dire: *Ballistics* or *Artillerie* problem. There are some possibilities to contribute this new way of historiography. And this study does not only concerned history of the exact science but also more general situation of Central Europe in the half of 18th Century: Military, Modern State, mathematician in the society, international transition of new devices.

2. What is Ballistics?

Firstly I underline the origin of word. It came from an ancient Greek verb, Βάλλειν that has meaning of *to throw*. Simply, that study had treated the exact trajectory of

something to be thrown from the Roman Ages, but it became a compound domain of military problem and has been called one sort of *Mixed Mathematics*. It is an archaic word that was used until the end of 18th Century very frequently. After knowing this word, we must not regard confrontation between pure and mixed mathematics as pure and the difference between pure and applied one nowadays[1]. So, modern ballistics has 4 types of contain: Internal, Transition, External, Terminal ballistic. More precisely the archaic study in 18th Century was discussing Internal and one part of Transition are.

Ballistics had changed its meaning from theory to experimental sciences. Until the end of 17th Century, Gerolamo Cardano and Galileo Galilei had argued the projectile theory but, in this age, the theoretical aspect of ballistic problem had been too minimized. Especially Galileo had been satisfied with the trajectory parabola about this too difficult curve[2].

The study in16th Century was too simple. Does High speed projectile truly draw a idealized parabolic curve? So as to answer this problem, we needed to wait the research of Isaac Newton.

3. Newton's *Principia* (1726, 3rd ed.) as a Philosophical Beginning

Firstly, we note one phrase of *Principia*, the 3rd vol.

In the preceding books I have laid down the principles of philosophy; principles not philosophical but mathematical; such, namely, as we may build our reasoning upon in philosophical inquiries. These principles are the laws and conditions of certain motions, and powers and forces, which chiefly have respect to philosophy; but lest they should have appeared of themselves dry and barren, *I have illustrated them here and there with some philosophical scholiums, giving an account of such things as are of more general nature, and which philosophy seems chiefly to be founded on;* such as the density and the resistance of bodies,

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[1] There is a more detailed explication of the difference in *Kagaku Academy to Yuyona Kagaku* (2011), 7th Chapter, by Sayaka OKI in Japanese. She treats the argument of political arithmetic there. In French *Encyclopédie,* Mixed Mathematics has been treated as broader meaning including Social and States problems.

[2] Galileo, *Discorsi e Dimostrazioni Matematiche Intorno a Due Nuove Scienze* (1638), the 4th day.

spaces void of all bodies, and the motion of light and sounds. (above *Principia*, p. 386, trans. By Florian Cajori, California, 1934. I underlined the important points in using Italic font. These fonts are not existed in the original Text.)

Here I analyze the detailed state and content of ballistics in *Principia.* Generally, in 3rd Book, Newton discussed the system of the world in popular language like *ut a pluribus legeretur*[3]. But more mature consideration was needed for understanding this part. Because it is for those who had first mastered the earlier 2 books. It is not until the readers understood them that they can enter into this applied problem. Even for Newton, this ballistic problem needed a careful and complete understanding of New Mechanics. A physical point of view, Newton explained that air resistance is simply proportion to a square of flying object's velocity. The Modification about this simple description became a key to new ballistics in 18th Century.

4. Movement of Continent at the beginning of 18th Century

Here we must recognize earlier study of ballistics in the Continent. Important Person is Jacques Cassini (1677-1756), son of Giovanni Domenico Cassini, successor of Observatoire de Paris. His famous work is *Traité de la grandeur et de la figure de la terre* (1720) that was focused on the size estimating problem of the Earth, really frequently treated in this époque.

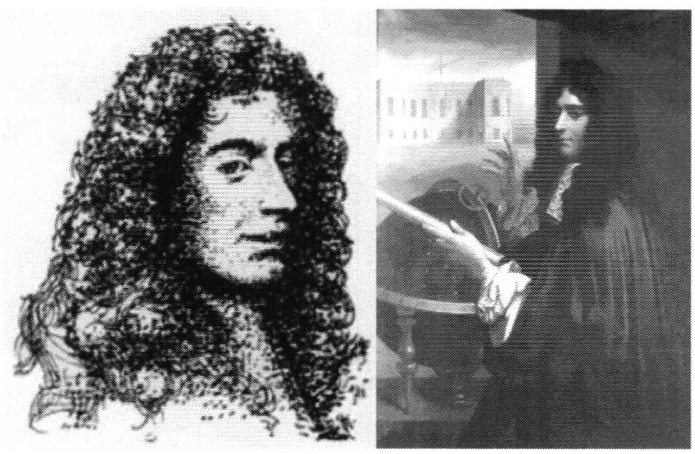

Fig.1. Jacques Cassini (left) and Giovanni Domenico Cassini (right)

[3] Eng. Translation: As it is [able to be] read by many people.

Jacques Cassini's new device has some remarkable features and it has common to the *Ballistic Pendulum* by B. Robins, later known. On the other word, it can be seen as French version of Ballistic Pendulum. It was taken up to a problem in Académie Royale des Sciences. In those days, J. Cassini was a core member of French Academy. But this tradition of study disappeared suddenly in the beginning of 18th Century. We can verify it by the comparaison of descriptions of *l'Encyclopédie* (1751-72) with *l'Encyclopédie méthodique*. This disappear and revive in later Robins' work is a truly enigmatical problem.

So, I discussed the structure of J. Cassini's device technically. It was presented in the conference of French Academy in 1707. Its structure consists of T style timbering which is attached to one strong metal ``D part'' and used in order to measure the oscillation of D part. It can express a force of the impact of attacked flying body. It has primitive and small structure but is enough to measure the speed of bullet.

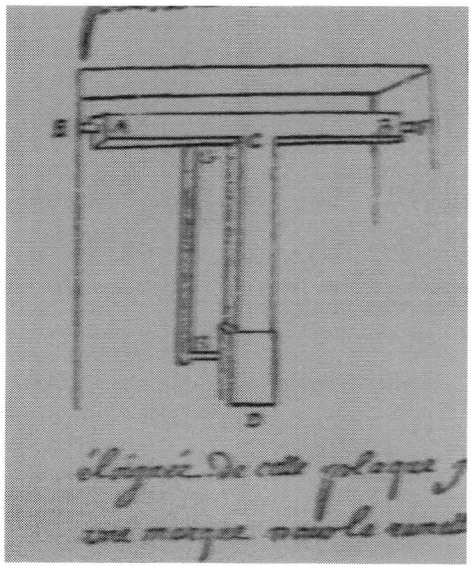

Fig.2. J. Cassini Device

5. *New Principles* by B. Robins

De hors de Cassini's invention, new suggest and approach to measuring the velocity of bullet has been created in England. That is *New Principle of Gunnery* (1746) written by Benjamin Robins (1707-1751). This book has several important and remarkable points, but especially a modification of Newton's mathematical assertion. Robins has denied it like this: resistance of air is not always proportional to the square of the speed of attacked body. Particularly, in the neighbor speed of Mach 1 and supersonic, a resistance value can be changed drastically. We will return on the problem after showing Robins' life.

6. Benjamin Robins and his character

He is a technician of British East India Company and pious Newtonian, a mathematically gifted person. His main concerning in this age was measuring air resistance when the speed of flying body passed sound of speed, ``supersonic''. As I had already noted that, he insisted on the incorrectness about the former theory of projectile in *Principia*, $r \propto v^2$. He has invented, that is called, ``Ballistic Pendulum and its function is in order to determine the nozzle velocity of a bullet. Its structure is quite resembled to J. Cassini's one. Firstly, a sturdy lead plate is fixed in bottom of machine. The strength of detachable plate is valuable. Secondly, its utilization is like that: to set up nearby the gun barrel and measure the oscillation after shooting.

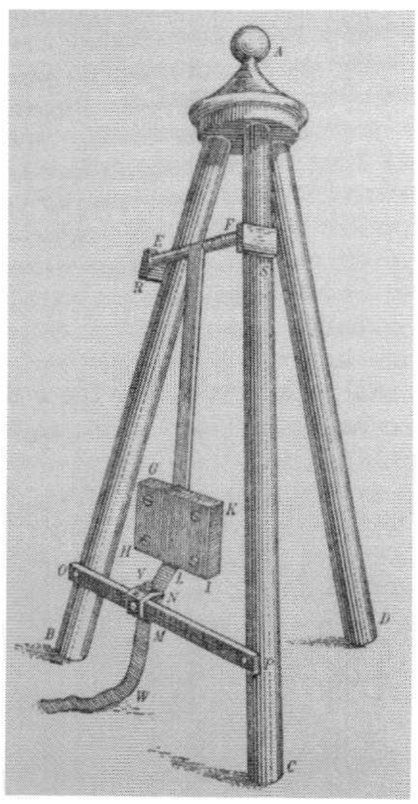

Fig. 3. Robins' Ballistic Pendulum

After inventing the instrument, Robins researched the change of bullet speed inside of zone of Canon evidentially. His mathematical style of *New Principle of Gunnery* is a naturally traditional one.

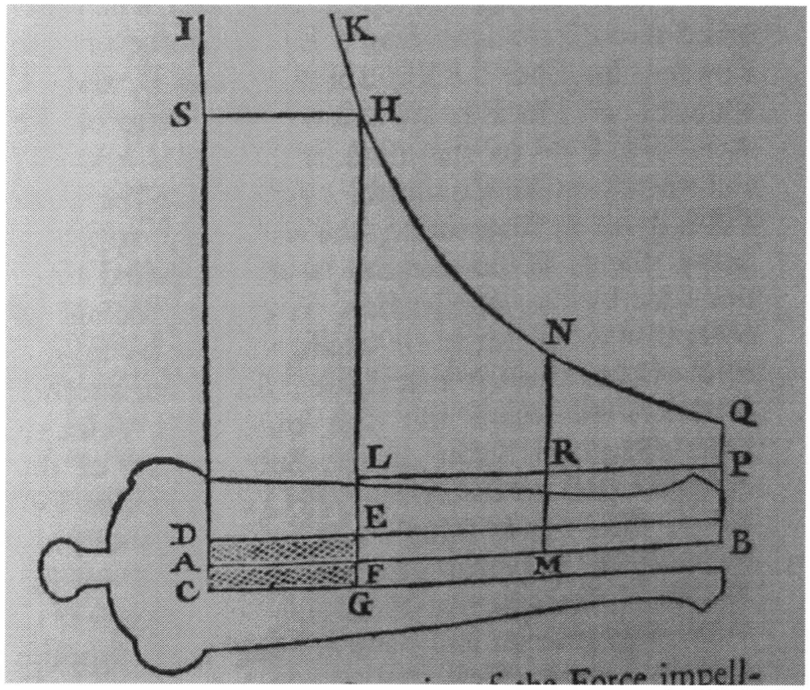

Fig. 4. Prop. 7, Chap.1 in *New Principle of Gunnery*

In Fig. 4, I show an example of inner ballistics problem here in order to represent the decreasing of a nozzle pressure. He persisted in the archaic style here, 7th Proposition in 1st Chapter. At first, a bullet has started from the base DAC to point B Via F and M while a hyperbolic curve KHQ represents a state of inner pressure. Robins' originalities and some tricks are like these 5 points:

1. Application and renovation of Principia 1-7. Prop. 39, concerning the resistance of air against flying object.
2. More precisely, division of resistant coefficient of high speed that is more than 1700 ft. /s from law speed body when treating a moving body through resistant media.
3. Division of inner ballistics from exterior one when a bullet reached or nearly reached sonic speed.
4. Precise determination of nozzle velocity by renovating Ballistic Pendulum[4].

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[4] Robins uses the word, *first velocity* in original text.

5. Introduction of `*air column*' and quantitative approach on the measure of air resistance.

Robins' approach was completely new one and we can make a distinguish line from the research of 16th and17th Century.

7. L. Euler's new translation and Theoretical Innovation

After Robins' new work, translation into German was projected by Leonhard Euler. German version's name is *Neue Grundsätze der Artillerie* (1745)[5]. Euler has just arrived at Berlin in 1741, and he began to insist on the importance of Robins' work to Prussian King, Frederick the 2nd (1712-1786)[6]. He praised Robins' contribution but was discontented with Robins' method. He felt that one needed analytical rewriting and he started it up. Euler's description and rewriting has several features.

1. His work is not only an additional but also a completely rewriting into analytical one. In the 1st Book, Prop. 6, he argued the orbit of bullet after leaving out from a nozzle. Nowadays, it belongs to Exterior Ballistics Problem[7].
2. Basic character of his Analysis is compound of a simple propositions' group.
3. From careful consideration of the case *horizontal projectile* of a supersonic flying object[8].

And then, numerical analysis that Euler has adopted is like these, *b*: falling altitude corresponding at nozzle velocity, *h*: coefficient 28845 feet, *c*: diameter of bullet, n: relative density between bullet and air.

$$v = b - \frac{3b(b+h)x}{4nc} + \frac{9b(b+h)(2b+h)x^2}{32n^2c^2h^2}.$$

From this result, third *Anmerkung*, `annotation' starts in adding these inclinatory conditions. χ: horizontal distance, φ: degree of shot, *A, B, C*: coefficients.

5 Eng. Translation: *New Principles of Gunnery*, just as same as original one.

6 His German common name `Friedrich der Große' is came from his character as an Enlightened Despot.

7 [Euler 1745]; EOO II 14. P.366.

8 After drawing the horizontal component, it seems to nearly straight line. *Ibid.*

$$\chi = +a\tan\varphi + \frac{A}{\cos\varphi - B\varphi} + B\tan\varphi + \frac{C(1+\cos\varphi^2)}{\cos\varphi}$$

$$-a\tan\varphi - \frac{A}{\cos\varphi + B\varphi} - B\tan\varphi - \frac{C(1+\cos\varphi^2)}{\cos\varphi} + \cdots$$

From this equation, lead to the best high range can be computed[9].

8. Possibilities of relationship with hydrodynamics in 18th Century (1)

We argued Robins' and Euler's systematic and not brief work. But there are other works about general problem about the hydrodynamics in this age. One Euler's article, [E226] *Principes generaux du mouvement des fluides*, presented in 1755, and published in 1757 is, as we know, the first recommendation of famous Euler Equation about hydrodynamics[10]. After this research, he had two possibilities in order to treat ballistic problem: Euler equation of hydrodynamics or a former empirical equation. It is a really important that common and structural limit of hydrodynamics study in 18th Century was concerning about the concept of *Viscosity*.

Even the most powerful mathematicians and physicists could not think hydromechanics' problems without the condition of ideal fluid. Critical mathematical defeats are treatises of trigonometric function and boundary condition because they are needed for the purpose of solving partial differential equations. So, here, I briefly compare the difference between the Euler Equation and the Navier-Stokes one in 19th Century[11].

Firstly, *i* component of force of viscosity for a unit volume,

$$\frac{\partial F_{ik}}{\partial x_k} = \eta\frac{\partial}{\partial x_k}\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right) = \eta\left(\nabla^2 v_i + \frac{\partial}{\partial x_i}\mathrm{div}v\right),$$

then with an only consideration of no contractive fluid and eliminating div *v*.

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9 *Ibid.* p. 391.

10 Eng. Translation: *General Principles of floats' movement.*

11 Claude-Louis Navier (1785-1863) was a French engineer and physicist who specialized in hydrodynamics. The famous Navier–Stokes equations are named after him and George Gabriel Stokes (Irish, 1819-1903) .

Therefore, the equation needed is:

$$\rho\left\{\frac{\partial v}{\partial t}+(v\cdot\nabla)v\right\}=-\nabla P+\eta\nabla^2 v+\rho f.$$

And then, the Navier-Stokes Equation was finally derived:

$$\frac{\partial v}{\partial t}+(v\cdot\nabla)v=-\frac{1}{\rho}\nabla P+\nu\nabla^2 v+f.$$

The difference between this one and Euler's old one of fluid is only the existence of terms of *viscosity*. This point is an important *Merkmal* that can divide hydrodynamics study in 18th from newer one. In turn to ballistic problem, should we consider it a merely experimental law? We could treat it as a milestone for the accomplishment and application of a more sophistical equation in the domain of hydrodynamics. We need more systematic and detailed research about this bridge between empirical problem and theoretical problem.

9. Conclusion

So, we can here note several important conclusions. Firstly, new attempt on ballistics research has started up from Newton's inheritance and it has rooted from some philosophical states, but it needed a more mature situation about mathematics of the half of 18th Century although the existence of J. Cassini's interesting approach. Secondly, in the 3rd generation from Newton, Leibniz, ballistics problem has become communicative and political issue in order to satisfy Modern State demand, especially Prussia's case. Fusion of two groups knowledge has already been promoted through new translation and analytical rewriting[12].

Before the complement of Euler Equation in 18th Century, we need verify if there were some influences from ballistic problem. It has possibility to be remained chiefly at the position of experimental law. In same time, ballistic problem contains mathematically and technically more severe condition and difficulty: it could have opened the door to the hydrodynamics study in 19th Century.

[12] One cannot recognize the reaction of British mathematicians after Euler's translation.

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